

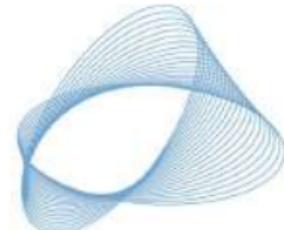
Conformal Renormalization and Energy Functionals in AdS gravity

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Outline

Renormalization of bulk/surface functionals

Conformal renormalization in AdS gravity

Conformal renormalization in curved backgrounds

Conformal renormalization in curved backgrounds

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Renormalization of bulk/surface functionals

• Renormalization of the bulk functional

• Renormalization of the surface functional

• Renormalization of the total functional

Outline

Renormalization of bulk/surface functionals

Alternative Renormalization Scheme: Kounterterms

Conformal and renormalized energy functionals

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Conformal renormalization in AdS gravity

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Counterterms and Kounterterms in AdS gravity

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Conformal energy functionals in AdS gravity

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Beyond Kounterterms: Conformal Renormalization

Conformal Energy Functionals in AdS gravity

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Conformal Renormalization and Energy Functionals in AdS gravity

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Counterterms and Kounterterms in AdS gravity

Beyond Kounterterms: Conformal Renormalization

Renormalization of Codimension-2 Functionals

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Counterterms and Kounterterms in AdS gravity

Beyond Kounterterms: Conformal Renormalization

Renormalization of Codimension-2 Functionals

Black Hole Thermodynamics in AdS gravity

Euclidean static black hole metric

$$ds^2 = f^2(r)d\tau^2 + \frac{dr^2}{f^2(r)} + r^2 d\Omega_{D-2}^2, \quad f^2(r) = 1 - \frac{2\omega_D GM}{r^{D-3}} + \frac{r^2}{\ell^2}$$

Euclidean action

$$I_{BH} = \frac{1}{16\pi G} \int d^D r \sqrt{-g} (R - 2\Lambda), \quad \Lambda = -\frac{(D-1)(D-2)}{2r^2}$$

Temperature

$$T I_{bulk}^E = \frac{(D-3)}{(D-2)} M - TS + \lim_{S \rightarrow \infty} \frac{V(S^{D-2}) r^{D-1}}{S^{D-2}}$$

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Temperature

$$T_{BH} = \frac{1}{4\pi r_{+}} \sqrt{f'(r_{+})} \sqrt{f(r_{+})}, \quad A = \frac{(D-1)(D-2)}{2\pi r_{+}^{D-2}}$$

Entropy

$$S_{BH}^E = \frac{(D-3)}{(D-2)} M - TS + \lim_{R \rightarrow \infty} \frac{V(S^2 \rightarrow) r^{D-1}}{8\pi G T}$$

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Einstein-AdS gravity

$$I_{EH} = \frac{1}{16\pi G} \int_M d^D x \sqrt{-g} (R - 2\Lambda), \quad \Lambda = -\frac{(D-1)(D-2)}{2\ell^2}$$

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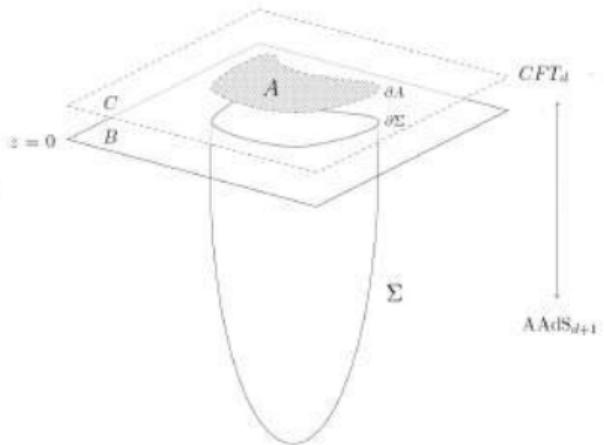
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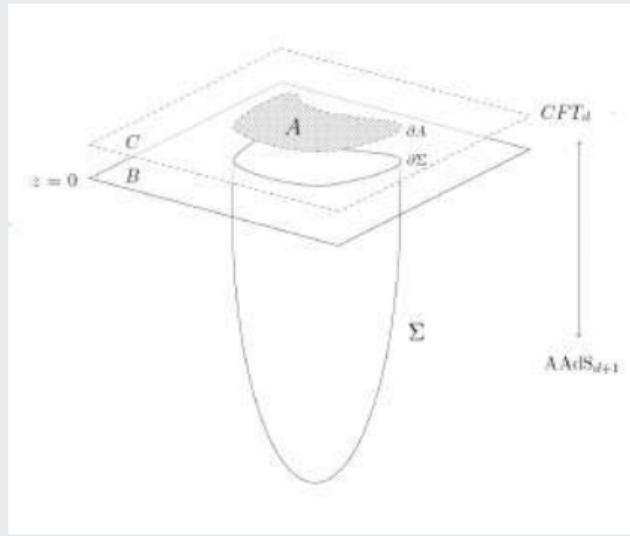
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Holographic Entanglement Entropy from Minimal Surface [Ryu-Takayanagi, 2006]



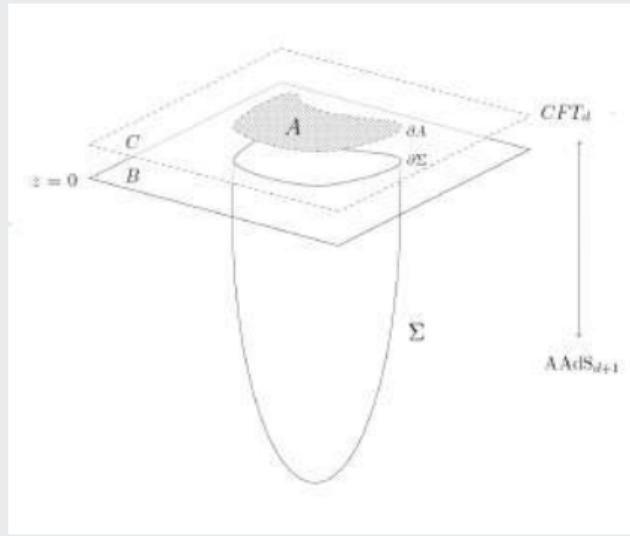
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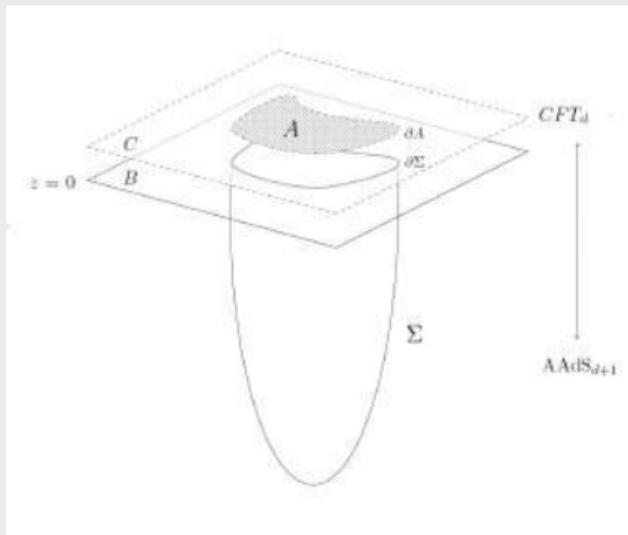


HEE from a Cosmic Brane [Lewkowycz-Maldacena, 2013]

$$S = -\partial_\alpha \lim_{\alpha \rightarrow 1} I_{\text{grav}}^{(\alpha)}$$

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Energy functionals

Bulk gravity action evaluated in conical defects

$$I_{\text{EH}}^{(\alpha)} = \frac{1}{16\pi G} \int_{M^{(\alpha)}} d^4x \sqrt{g} R^{(\alpha)} = \frac{1}{16\pi G} \int_M d^4x \sqrt{g} R + \frac{(1-\alpha)}{4G} \mathcal{A}[\Sigma]$$

$$\mathcal{A}[\Sigma] = \int_{\Sigma} d^2y \sqrt{\gamma}$$

$$S = \frac{1}{G} I_{\text{EH}}^{(\alpha)}$$

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Counterterms in AdS gravity

Holographic Renormalization [Henningson and Skenderis, 1998]

$$I_{\text{ren}} = I_{EH} - \frac{1}{8\pi G} \int_M d^d x \sqrt{-h} K + \int_M d^d x L_{ct}(h, \mathcal{R}, \nabla \mathcal{R})$$

where the counterterm is given by

$$\begin{aligned} 8\pi G L_{ct} = & \frac{2}{d} \sqrt{-h} + \frac{\delta \sqrt{h}}{2(d-2)} R + \frac{\delta^2 \sqrt{h}}{2(d-2)^2(d-3)} \left(R^{ij} R_{ij} - \frac{d}{d-1} R^2 \right) \\ & + \frac{\delta^3 \sqrt{h}}{(d-3)(d-2)(d-1)} \left(\frac{d-2}{d-1} R R^{ij} R_{ij} - \frac{d(d+2)}{16(d-2)^2} R^3 \right. \\ & \left. - 2R^{ij} R^{kl} R_{ijkl} - \frac{d}{16(d-2)} \nabla_i R \nabla^i R + \sqrt{h} \nabla_i R_{ij} \right) + \dots \end{aligned}$$

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$$\begin{aligned} 8\pi G L_0 &= \frac{1}{2} \sqrt{-h} \left[R + \frac{d(d-1)}{2(d-2)} \left(R^2 R_0 - \frac{d-2}{d-3} R^2 \right) \right. \\ &\quad \left. - \frac{d(d-1)}{2(d-2)} \left(\frac{d-2}{d-3} R R_0 R_0 + \frac{d(d-2)}{16(d-3)} R^2 \right) \right] \\ &\quad - \frac{1}{2} \left(d(d-1) R_0^2 - \frac{d(d-2)}{16(d-3)} R^2 R_0 + \frac{d(d-2)}{16(d-3)} R^2 R_0 \right) \end{aligned}$$

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Counterterms [Balasubramanian-Kraus, 1999], [Emparan, Johnson, Myers, 1999]

$$\begin{aligned} 8\pi G L_{ct} = & \frac{d-1}{\ell} \sqrt{-h} + \frac{\ell \sqrt{-h}}{2(d-2)} \mathcal{R} + \frac{\ell^3 \sqrt{-h}}{2(d-2)^2(d-4)} \left(\mathcal{R}^{ij} \mathcal{R}_{ij} - \frac{d}{4(d-1)} \mathcal{R}^2 \right) \\ & + \frac{\ell^5 \sqrt{-h}}{(d-2)^3(d-4)(d-6)} \left(\frac{3d-2}{4(d-1)} \mathcal{R} \mathcal{R}^{ij} \mathcal{R}_{ij} - \frac{d(d+2)}{16(d-1)^2} \mathcal{R}^3 \right. \\ & \left. - 2 \mathcal{R}^{ij} \mathcal{R}^{kl} \mathcal{R}_{ijkl} - \frac{d}{4(d-1)} \nabla_i \mathcal{R} \nabla^i \mathcal{R} + \nabla^k \mathcal{R}^{ij} \nabla_k \mathcal{R}_{ij} \right) + \dots \end{aligned}$$

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Black Hole Thermodynamics and Counterterms

Counterterm Method reproduces BH Thermo

$$G = U - TS$$

Maxwell's Eqns

$$T = M + E_0$$

Maxwell's Eqns

$$E_0 = (-)^{\frac{1}{2}} \left(\frac{2\pi e^2}{(2\pi)^2} \frac{N_0}{N_0 + 1} \right) \frac{e^2}{r^2}$$

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Internal Energy

$$U = M + E_0$$

Temperature

$$T = \frac{(-)^{\frac{D-2}{2}}(n-1)W^{\frac{1}{2}}\lambda^{D-2}}{(2\pi)^{\frac{D-2}{2}}\Gamma(\frac{D-2}{2})}$$

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Vacuum Energy in $D = 2n + 1$ dimensions

$$E_0 = (-1)^n \frac{(2n-1)!!^2}{(2n)!} \frac{\text{Vol}(S^{2n-1})}{8\pi G} \ell^{2n-2}$$

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Kounterterms in AdS gravity

Extrinsic counterterms

$$\tilde{I}_{ren} = I + c_d \int_{\partial M} d^d x B_d(h, K, \mathcal{R})$$

Kounterterms = counterterms of unusual sort (depend on K_{ij} and $\mathcal{R}_{ij}^{kl}(h)$)

Example: B_{2n+1}

$$B_{2n+1} = 2\pi\sqrt{-h} \int_0^\infty dt \, d^{2n+2p+1}x \, K^t \left(\frac{1}{2} R_{ijkl}^{(p)} - t^2 K_{ij}^k K_{il}^j \right) \times \dots \\ \dots \times \left(\frac{1}{2} R_{ijkl}^{(p)} - t^2 K_{ij}^k K_{il}^j \right)$$

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Example: $B_{d=1}$

$$B_{d=1} = 2\pi\sqrt{-h} \int_0^{\infty} dt \delta^{(d-2)}(t) K_t \left(\frac{R_{tt}}{2} - t^2 K_t^2 K_t \right) + \dots$$

$$\dots + \left(\frac{R_{tt}}{2} - t^2 K_t^2 - K_t^4 \right)$$

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Example:

$$B_{d=1} = 2\pi\sqrt{-h} \int d^d x \sqrt{-h} \left(R^{ij} - F^{ijkl}K_{ij} \right) \dots$$

$$\dots \rightarrow \left(R^{ij} - F^{ijkl}K_{ij} - F^{ijkl}K_{jl} \right)$$

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$D = 2n$ dimensions [R.O., hep-th/0504233]

$$B_{2n-1} = 2n\sqrt{-h} \int_0^1 dt \delta_{[j_1 \dots j_{2n-1}]}^{[i_1 \dots i_{2n-1}]} K_{i_1}^{j_1} \left(\frac{1}{2} \mathcal{R}_{i_2 i_3}^{j_2 j_3} - t^2 K_{i_2}^{j_2} K_{i_3}^{j_3} \right) \times \dots \\ \dots \times \left(\frac{1}{2} \mathcal{R}_{i_{2n-2} i_{2n-1}}^{j_{2n-2} j_{2n-1}} - t^2 K_{i_{2n-2}}^{j_{2n-2}} K_{i_{2n-1}}^{j_{2n-1}} \right)$$

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Kounterterms in AdS gravity

Kounterterms in $D = 2n + 1$ [R.O., hep-th/0610230]

$$\begin{aligned}
 B_{2n} &= 2n\sqrt{-h} \int_0^1 dt \int_0^t ds \delta_{[i_1 \dots i_{2n}]}^{[j_1 \dots j_{2n}]} K_{j_1}^{i_1} \delta_{j_2}^{i_2} \left(\frac{1}{2} \mathcal{R}_{j_3 j_4}^{i_3 i_4} - t^2 K_{j_3}^{i_3} K_{j_4}^{i_4} + \frac{s^2}{\ell^2} \delta_{j_3}^{i_3} \delta_{j_4}^{i_4} \right) \times \dots \\
 &\quad \dots \times \left(\frac{1}{2} \mathcal{R}_{j_{2n-1} j_{2n}}^{i_{2n-1} i_{2n}} - t^2 K_{j_{2n-1}}^{i_{2n-1}} K_{j_{2n}}^{i_{2n}} + \frac{s^2}{\ell^2} \delta_{j_{2n-1}}^{i_{2n-1}} \delta_{j_{2n}}^{i_{2n}} \right).
 \end{aligned}$$

Kounterterms in AdS gravity

In $D = 2n$ dimensions

$$\begin{aligned}\text{tr}(F^n) &= dL_{2n-1}^{CS}(A) \\ F &= dA + A \wedge A\end{aligned}$$

Explicit realization of Chern-Simons forms

$$L_{2n-1}^{CS}(A) = n \int_0^1 dt \text{tr} [AF_t^{n-1}] \quad F_t = t dA + t^2 A^2$$

$$\int_M (\text{Euler})_{2n} = (dx)^n n! \chi(M_{2n}) + \int_{\partial M} B_{2n-1}$$

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Global issues (topology)

$$\int_{M_{2n}} (\text{Euler})_{2n} = (4\pi)^n n! \chi(M_{2n}) + \int_{\partial M_{2n}} B_{2n-1}$$

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Kounterterms in AdS gravity

$D = 2n + 1$ dimensions

$$L_{2n+1}^{TF}(A, \bar{A}) = (n+1) \int_0^1 dt \text{tr} [(A - \bar{A}) F_t^n]$$

$$F_t = dA_t + A_t^2, \quad A_t = tA + (1-t)\bar{A}$$

Gauge-invariant extension of CS forms $L_{2n+1}^{TF}(A, \bar{A}) = L_{2n+1}^{CS}(A) - L_{2n+1}^{CS}(\bar{A}) + d\beta_{2n}(A, \bar{A})$

$\beta_{2n}(A, \bar{A})$

$$\beta_{2n}(A, \bar{A}) = \int_0^1 dt \int ds \text{tr} [A_t (A - \bar{A}) F_s^{n-1}]$$

$$F_s = d\tilde{A}_s + s(s-1) A_s^2$$

Kounterterms in AdS gravity

$D = 2n + 1$ dimensions

$$L_{2n+1}^{TF}(A, \bar{A}) = (n+1) \int_0^1 dt \text{tr} [(A - \bar{A}) F_t^n]$$

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Kounterterms in AdS gravity

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Contact term

$$\begin{aligned} \beta_{2n}(A, \bar{A}) &= \int_0^1 dt \int_0^t ds \operatorname{tr} [A_t (A - \bar{A}) F_{st}^{n-1}] \\ F_{st} &= sF_t + s(s-1)A_t^2 \end{aligned}$$

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Gauge-invariant extension of CS forms $L_{2n+1}^{TF}(A, \bar{A}) = L_{2n+1}^{CS}(A) - L_{2n+1}^{CS}(\bar{A}) + d\beta_{2n}(A, \bar{A})$

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Renormalized Action = Renormalized Volume

Black Hole Thermodynamics

$$TI_{bulk}^E = \frac{(D-3)}{(D-2)}M - TS + \lim_{r \rightarrow \infty} \frac{V(S^{D-2})}{8\pi G} \frac{r^{D-1}}{\ell^2}$$

Incorrect Black Hole Thermo

$$T_{bulk} \int R_1 = \frac{M}{(D-2)} + E_0 - \lim_{r \rightarrow \infty} \frac{V(S^{D-2})}{8\pi G} \frac{r^{D-1}}{\ell^2}$$

Correct Black Hole Thermo with $U = M + E_0$

Renormalized Action = Renormalized Volume

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$$TI_{bulk}^E = \frac{(D-3)}{(D-2)}M - TS + \lim_{r \rightarrow \infty} \frac{V(S^{D-2})}{8\pi G} \frac{r^{D-1}}{\ell^2}$$

From the previous slide

$$T r_{\infty} \int \mathcal{R}_1 = \frac{M}{(D-2)} + B_0 - \lim_{r \rightarrow \infty} \frac{V(S^{D-2})}{8\pi G} \frac{r^{D-1}}{\ell^2}$$

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Euclidean Kounterterms

$$T c_d \int_{\partial M} B_d = \frac{M}{(D-2)} + E_0 - \lim_{r \rightarrow \infty} \frac{V(S^{D-2})}{8\pi G} \frac{r^{D-1}}{\ell^2}$$

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Correct Black Hole Thermo with $U = M + E_0$

Boundary conditions in AdS gravity

Fefferman-Graham expansion for AAdS Einstein spaces

$$ds^2 = \frac{\ell^2}{z^2} dz^2 + \frac{1}{z^2} g_{ij}(x, z) dx^i dx^j, \quad g_{ij}(x, \rho) = g_{(0)ij}(x) + z^2 g_{(2)ij}(x) + \dots$$

Boundary

at $z = 0$

$$g_{ij} = g_{(0)ij}$$

Conformal boundary condition

$$\mathcal{W}_{\text{ext}} = \frac{1}{2} \int \sqrt{-g_0} T^{00} g_{00} d\Omega_0$$

Boundary conditions in AdS gravity

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Dirichlet b.c. $\delta h_{ij} = 0$ does not make sense in AAdS spaces

[Papadimitriou and Skenderis, 2004]

$$h_{ij} = \frac{g_{(0)ij}}{z^2} + \dots$$

Renormalization = variational problem in $g_{(0)ij}$

$$\delta I_{ren} = \frac{1}{2} \int_{\partial M} \sqrt{-g_{(0)}} T^{ij}[g_{(0)}] \delta g_{(0)ij}$$

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Kounterterms and Holography

Asymptotic form of the extrinsic curvature

$$K_{ij} = \frac{1}{\ell} \frac{g(0)ij}{z^2} + \dots$$

$$\tilde{I}_{ren} = I_{ren} + \alpha_1 \int d\sigma D(f(\sigma)) K$$

Conformal counterterm

$$\delta \tilde{I}_{ren} = \frac{1}{2} \int \sqrt{-g(0)} \tau^{ij} \delta g(0) ij$$

Kounterterms and Holography

Asymptotic form of the extrinsic curvature

$$K_{ij} = \frac{1}{\ell} \frac{g_{(0)ij}}{z^2} + \dots$$

$$\tilde{L}_{\text{int}} = L_{\text{ext}} + \alpha \int d^d x R(f(\rho)) K$$

where $f(\rho)$ is a smooth function

$$\delta \tilde{L}_{\text{int}} = \frac{1}{2} \int d^d x \sqrt{-g_{(0)}} \tau^{ij} \delta g_{(0)ij}$$

Kounterterms and Holography

Asymptotic form of the extrinsic curvature

$$K_{ij} = \frac{1}{\ell} \frac{g_{(0)ij}}{z^2} + \dots$$

Counterterms of a different sort...

$$\tilde{I}_{ren} = I_{EH} + c_d \int_{\partial M} d^d x B(f(h), K)$$

...as long as the theory is *holographic*

$$\delta \tilde{I}_{ren} = \frac{1}{2} \int_{\partial M} \sqrt{-g_{(0)}} \tau^{ij} \delta g_{(0)ij}$$

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From extrinsic to intrinsic renormalization in 4D

AdS gravity action + KTs

$$\tilde{I}_{\text{ren}} = I_{EH} + \frac{\ell^2}{16\pi G} \int_{\partial M} d^3x \sqrt{-h} \delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K_{i_1}^{j_1} \left(\frac{1}{2} \mathcal{R}_{i_2 i_3}^{j_2 j_3}(h) - \frac{1}{3} K_{i_2}^{j_2} K_{i_3}^{j_3} \right).$$

Adding zero...

$$\tilde{I}_{\text{ren}} = I_{EH} - \frac{1}{8\pi G} \int_{\partial M} d^3x \sqrt{-h} K + \int_{\partial M} d^3x L_{ct}.$$

$$L_{ct} = \frac{\ell^2}{16\pi G} \sqrt{-h} \delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K_{i_1}^{j_1} \left(\frac{1}{2} \mathcal{R}_{i_2 i_3}^{j_2 j_3}(h) - \frac{1}{3} K_{i_2}^{j_2} K_{i_3}^{j_3} + \frac{1}{\ell^2} \delta_{i_2}^{j_2} \delta_{i_3}^{j_3} \right).$$

Final form of the action:

$$K_j^i - \frac{1}{\ell^2} \delta_j^i - \ell S_j^i(h) + \mathcal{O}(\ell^2), \quad S_j^i(h) = \frac{1}{d-2} (\mathcal{R}_j^i(h) - \frac{1}{2(d-1)} \delta_j^i \mathcal{R}(h))$$

From extrinsic to intrinsic renormalization in 4D

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Adding zero...

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$$L_{ct} = \frac{\ell^2}{16\pi G} \sqrt{-h} \delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K_{i_1}^{j_1} \left(\frac{1}{2} \mathcal{R}_{i_2 i_3}^{j_2 j_3}(h) - \frac{1}{3} K_{i_2}^{j_2} K_{i_3}^{j_3} + \frac{1}{\ell^2} \delta_{i_2}^{j_2} \delta_{i_3}^{j_3} \right).$$

From now on we will drop the subscript "ct".

$$K_j^i = \frac{1}{d-2} \delta_j^i - \epsilon S_j^i(h) + \mathcal{O}(R^2), \quad S_j^i(h) = \frac{1}{d-2} (\mathcal{R}_j^i(h) - \frac{1}{2(d-1)} \delta_j^i R(h))$$

From extrinsic to intrinsic renormalization in 4D

AdS gravity action + KTs

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$$K_i^j = \frac{1}{2} \delta_{ij} - \mathcal{S}_i(h) + O(R^2), \quad \mathcal{S}_i(h) = \frac{1}{d-2} (R_i(h) - \frac{1}{2(d-2)} \delta_{ij} R(h))$$

From extrinsic to intrinsic renormalization in 4D

AdS gravity action + KTs

$$\tilde{I}_{\text{ren}} = I_{EH} + \frac{\ell^2}{16\pi G} \int_{\partial M} d^3x \sqrt{-h} \delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K_{i_1}^{j_1} \left(\frac{1}{2} \mathcal{R}_{i_2 i_3}^{j_2 j_3}(h) - \frac{1}{3} K_{i_2}^{j_2} K_{i_3}^{j_3} \right).$$

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Fefferman-Graham expansion

$$K_j^i = \frac{1}{\ell} \delta_j^i - \ell S_j^i(h) + \mathcal{O}(\mathcal{R}^2), \quad S_j^i(h) = \frac{1}{d-2} (\mathcal{R}_j^i(h) - \frac{1}{2(d-1)} \delta_j^i \mathcal{R}(h))$$

From extrinsic to intrinsic renormalization in 4D

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From extrinsic to intrinsic renormalization

$$L_{ct} = \frac{\ell^2}{16\pi G} \frac{\sqrt{-g}}{z^3} \delta_{[j_1 j_2 j_3]}^{[i_1 i_2 i_3]} \left(\frac{\delta_{i_1}^{j_1}}{\ell} - \ell S_{j_1}^{i_1} \right) \times \\ \times \left(\frac{1}{2} \mathcal{R}_{i_2 i_3}^{j_2 j_3}(h) - \frac{1}{3} \left(\frac{\delta_{i_2}^{j_2}}{\ell} - \ell S_{j_2}^{i_2} \right) \left(\frac{\delta_{i_3}^{j_3}}{\ell} - \ell S_{j_3}^{i_3} \right) + \frac{1}{\ell^2} \delta_{i_2}^{j_2} \delta_{i_3}^{j_3} \right) + \dots$$

Conformal renormalization in AdS [D. Măntoiu and R.O., 2012, 2013]

$$L_\omega = \frac{1}{8\pi G} \frac{\sqrt{-g}}{z^3} \left(\frac{2}{\ell} + \frac{\ell}{2} \varphi^2 R(\varphi) \right) + \mathcal{O}(\varphi)$$

$$= \frac{1}{8\pi G} \frac{\sqrt{-g}}{z^3} \left(\frac{2}{\ell} + \frac{\ell}{2} \varphi^2 R(\varphi) \right)$$

From extrinsic to intrinsic renormalization

$$L_{ct} = \frac{\ell^2}{16\pi G} \frac{\sqrt{-g}}{z^3} \delta_{[j_1 j_2 j_3]}^{[i_1 i_2 i_3]} \left(\frac{\delta_{i_1}^{j_1}}{\ell} - \ell S_{j_1}^{i_1} \right) \times \\ \times \left(\frac{1}{2} \mathcal{R}_{i_2 i_3}^{j_2 j_3}(h) - \frac{1}{3} \left(\frac{\delta_{i_2}^{j_2}}{\ell} - \ell S_{j_2}^{i_2} \right) \left(\frac{\delta_{i_3}^{j_3}}{\ell} - \ell S_{j_3}^{i_3} \right) + \frac{1}{\ell^2} \delta_{i_2}^{j_2} \delta_{i_3}^{j_3} \right) + \dots$$

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Kounterterms turn into counterterms [O.Miskovic and R.O., 0902.2082]

$$L_{ct} = \frac{1}{8\pi G} \frac{\sqrt{-g}}{z^3} \left(\frac{2}{\ell} + \frac{\ell}{2} z^2 \mathcal{R}(g) \right) + \mathcal{O}(z) \\ = \frac{1}{8\pi G} \sqrt{-h} \left(\frac{2}{\ell} + \frac{\ell}{2} \mathcal{R}(h) \right).$$

From extrinsic to intrinsic renormalization

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$$L_{ct} = \frac{1}{8\pi G} \frac{\sqrt{-g}}{z^3} \left(\frac{2}{\ell} + \frac{\ell}{2} z^2 \mathcal{R}(g) \right) + \mathcal{O}(z) \\ = \frac{1}{8\pi G} \sqrt{-h} \left(\frac{2}{\ell} + \frac{\ell}{2} \mathcal{R}(h) \right).$$

Higher even dimensions

EH-AdS gravity +KTs

$$B_{2n-1} = 2n\sqrt{-h} \int_0^1 dt \delta^{[i_1 \cdots i_{2n-1}]}_{[j_1 \cdots j_{2n-1}]} K_{i_1}^{j_1} \left(\frac{1}{2} \mathcal{R}_{i_2 i_3}^{j_2 j_3} - t^2 K_{i_2}^{j_2} K_{i_3}^{j_3} \right) \times \cdots \\ \cdots \times \left(\frac{1}{2} \mathcal{R}_{i_{2n-2} i_{2n-1}}^{j_{2n-2} j_{2n-1}} - t^2 K_{i_{2n-2}}^{j_{2n-2}} K_{i_{2n-1}}^{j_{2n-1}} \right).$$

$$\tilde{I}_{\text{ren}} = I_{Dir} + \int_{\partial M} d^{2n-1}x L_{ct}$$

$$L_{ct} = c_{2n-1} B_{2n-1} + \frac{1}{2\pi G} \sqrt{-h} K \\ = \frac{(-t)^n}{n!} \sqrt{-h} \delta^{[i_1 \cdots i_{2n-1}]}_{[j_1 \cdots j_{2n-1}]} K_{i_1}^{j_1} \left[\left(\frac{1}{2} \mathcal{R}_{i_2 i_3}^{j_2 j_3} - P K_{i_2}^{j_2} K_{i_3}^{j_3} \right) \times \cdots \right. \\ \left. \cdots \times \left(\frac{1}{2} \mathcal{R}_{i_{2n-2} i_{2n-1}}^{j_{2n-2} j_{2n-1}} - P K_{i_{2n-2}}^{j_{2n-2}} K_{i_{2n-1}}^{j_{2n-1}} \right) + \frac{(-t)^n}{n!} \delta_{i_1}^{j_1} \cdots \delta_{i_{2n-1}}^{j_{2n-1}} \right].$$

Higher even dimensions

EH-AdS gravity +KTs

$$B_{2n-1} = 2n\sqrt{-h} \int_0^1 dt \delta^{[i_1 \cdots i_{2n-1}]}_{[j_1 \cdots j_{2n-1}]} K_{i_1}^{j_1} \left(\frac{1}{2} \mathcal{R}_{i_2 i_3}^{j_2 j_3} - t^2 K_{i_2}^{j_2} K_{i_3}^{j_3} \right) \times \cdots \\ \cdots \times \left(\frac{1}{2} \mathcal{R}_{i_{2n-2} i_{2n-1}}^{j_{2n-2} j_{2n-1}} - t^2 K_{i_{2n-2}}^{j_{2n-2}} K_{i_{2n-1}}^{j_{2n-1}} \right).$$

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Higher even dimensions

EH-AdS gravity +KTs

$$B_{2n-1} = 2n\sqrt{-h} \int_0^1 dt \delta^{[i_1 \cdots i_{2n-1}]}_{[j_1 \cdots j_{2n-1}]} K_{i_1}^{j_1} \left(\frac{1}{2} \mathcal{R}_{i_2 i_3}^{j_2 j_3} - t^2 K_{i_2}^{j_2} K_{i_3}^{j_3} \right) \times \cdots \\ \cdots \times \left(\frac{1}{2} \mathcal{R}_{i_{2n-2} i_{2n-1}}^{j_{2n-2} j_{2n-1}} - t^2 K_{i_{2n-2}}^{j_{2n-2}} K_{i_{2n-1}}^{j_{2n-1}} \right).$$

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Higher even dimensions

EH-AdS gravity +KTs

$$B_{2n-1} = 2n\sqrt{-h} \int_0^1 dt \delta^{[i_1 \dots i_{2n-1}]}_{[j_1 \dots j_{2n-1}]} K_{i_1}^{j_1} \left(\frac{1}{2} \mathcal{R}_{i_2 i_3}^{j_2 j_3} - t^2 K_{i_2}^{j_2} K_{i_3}^{j_3} \right) \times \dots \\ \dots \times \left(\frac{1}{2} \mathcal{R}_{i_{2n-2} i_{2n-1}}^{j_{2n-2} j_{2n-1}} - t^2 K_{i_{2n-2}}^{j_{2n-2}} K_{i_{2n-1}}^{j_{2n-1}} \right).$$

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$$L_{ct} = c_{2n-1} B_{2n-1} + \frac{1}{8\pi G} \sqrt{-h} K \\ = \frac{(-\ell^2)^n}{8\pi G (2n-2)!} \sqrt{-h} \delta^{[i_1 \dots i_{2n-1}]}_{[j_1 \dots j_{2n-1}]} K_{i_1}^{j_1} \int_0^1 dt \left[\left(\frac{1}{2} \mathcal{R}_{i_2 i_3}^{j_2 j_3} - t^2 K_{i_2}^{j_2} K_{i_3}^{j_3} \right) \times \dots \right. \\ \dots \times \left. \left(\frac{1}{2} \mathcal{R}_{i_{2n-2} i_{2n-1}}^{j_{2n-2} j_{2n-1}} - t^2 K_{i_{2n-2}}^{j_{2n-2}} K_{i_{2n-1}}^{j_{2n-1}} \right) + \frac{(-1)^n}{\ell^{2n-2}} \delta_{i_2}^{j_2} \dots \delta_{i_{2n-1}}^{j_{2n-1}} \right].$$

Higher even dimensions

EH-AdS gravity +KTs

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Higher even dimensions

Expanding and collecting...

$$L_{ct} = \frac{\sqrt{-h}}{8\pi G} \left[\frac{(2n-2)}{\ell} + \frac{\ell}{2(2n-3)} \mathcal{R} + \right. \\ \left. + \frac{\ell^3}{2(2n-3)^2(2n-5)} \left(2\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{(2n+1)}{4(2n-2)} \mathcal{R}^2 - \frac{(2n-3)}{4} \mathcal{R}^{ijkl}\mathcal{R}_{ijkl} \right) + \dots \right].$$

Boundary Weyl tensor $\mathcal{W}^{ijkl}\mathcal{W}_{ijkl}$ implies

$$\mathcal{R}^{ijkl}\mathcal{R}_{ijkl} = \mathcal{W}^{ijkl}\mathcal{W}_{ijkl} + \frac{4}{(2n-3)} (\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{1}{2(2n-2)} \mathcal{R}^2)$$

and obtain

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Higher even dimensions

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Higher even dimensions

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Holographic Renormalization = Kounterterms?

Well...almost. [G.Anastasiou, O.Miskovic, R.O. and I.Papadimitriou, 2003.06425]

$$\tilde{I}_{\text{ren}} = I_{\text{HR}} - \frac{\ell^3}{64\pi G(2n-3)(2n-5)} \int_{\partial M} \sqrt{-h} \mathcal{W}^{ijkl} \mathcal{W}_{ijkl} + \dots$$

A similar result in $D = 2n + 1$ dimensions.

Last term is zero for most AAdS spaces, but not for gravitational instantons.

Patching up the theory

$$I_{\text{HR}} = \tilde{I}_{\text{ren}} + \frac{\ell^3}{64\pi G(2n-3)(2n-5)} \int_{\partial M} \sqrt{-h} \mathcal{W}^{ijkl} \mathcal{W}_{ijkl} + \dots$$

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Renormalized AdS Action

Euler-Gauss-Bonnet Theorem in 4D

$$\int_{\partial M} d^3x B_3(K, \mathcal{R}) = \int_M d^4x GB - 32\pi^2 \chi(M)$$

Generalization to higher dimensions (Ricci scalar curvature)

$$I_{ren} = \frac{1}{16\pi G} \int d^d x \sqrt{-g} \left[(R - 2\Lambda) + \frac{\ell^2}{4} (R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4R_{\mu\nu} R^{\mu\nu} + R^2) \right]$$

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4D Renormalized AdS action [R. Aros et al, gr-qc/9909015]

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Renormalized AdS Action

GB coupling is also singled out by SUSY [Andrianopoli and D'Auria, arXiv:1405.2010]

$$I_{\text{ren}} = \frac{\ell}{208\pi G} \int d^D x \sqrt{-g} \delta_{\mu\nu}^{(D-m)} \left[R_{\mu_1\mu_2}^{(m)} + \frac{\delta_{\mu_1\mu_2}^{(m)}}{P} \right] \left[R_{\mu_3\mu_4}^{(m)} + \frac{\delta_{\mu_3\mu_4}^{(m)}}{P} \right].$$

$$W_{\mu}^{\nu} = R_{\mu}^{\nu} - 4S_{\mu}^{\lambda}\delta_{\nu}^{\lambda}, \quad \text{Schouten } S_{\mu}^{\nu} = \frac{1}{D-2}(R_{\mu}^{\nu} - \frac{1}{2(D-1)}\delta_{\mu}^{\nu}R)$$

Renormalized AdS Action

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$$I_{\text{GB}} = \frac{\ell}{20\pi G} \int d^D x \sqrt{-g} g^{\mu\nu} g^{\rho\sigma} \left[R_{\mu\rho} R_{\nu\sigma} - \frac{R^2 g_{\mu\nu} g_{\rho\sigma}}{D} \right] \left[R_{\mu\rho} R_{\nu\sigma} - \frac{R^2 g_{\mu\nu} g_{\rho\sigma}}{D} \right].$$

$$W_{\mu}^{\rho} = R_{\mu}^{\rho} - 4S_{\mu}^{\rho}d^2, \quad \text{Schouten } S_{\mu}^{\rho} = \frac{1}{D-2}(R_{\mu}^{\rho} - \frac{1}{2(D-1)}g_{\mu}^{\rho}R)$$

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Weyl tensor

$$W_{\mu\nu}^{\alpha\beta} = R_{\mu\nu}^{\alpha\beta} - 4S_{(\mu}^{\alpha} \delta_{\nu)}^{\beta}, \quad \text{Schouten } S_{\mu}^{\alpha} = \frac{1}{D-2} (R_{\mu}^{\alpha} - \frac{1}{2(D-1)} R^{\alpha}_{\mu} R)$$

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$$W_{\mu\nu}^{\alpha\beta} = R_{\mu\nu}^{\alpha\beta} - 4S_{[\mu}^{[a}\delta_{\nu]}^{\beta]}, \quad \text{Schouten } S_\mu^\alpha = \frac{1}{D-2}(R_\mu^\alpha - \frac{1}{2(D-1)}\delta_\mu^\alpha R)$$

Renormalized AdS Action

Weyl tensor for Einstein spaces $S_\mu^\alpha = -\frac{1}{2\ell^2} \delta_\mu^\alpha$

$$W_{(E)\mu\nu}^{\alpha\beta} = R_{\mu\nu}^{\alpha\beta} + \frac{1}{\ell^2} \delta_{[\mu\nu]}^{[\alpha\beta]}$$

Conformal and renormalized actions

$$I_{\text{con}} = \frac{\ell^2}{32\pi G} \int d^4x \sqrt{-g} R(x) \mu(x)$$

Renormalized AdS Action

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Conformal transformation of the metric

$$g_{\mu\nu} = \Omega^2 g_{\mu\nu}^0 = \Omega^2 (g_{\mu\nu} - g_{\mu\nu}^0 (x_{\mu\nu})) \Omega^{-2}$$

Renormalized AdS Action

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Renormalized action for Einstein spaces

$$I_{\text{ren}} = \frac{\ell^2}{64\pi G} \int_M d^4x \sqrt{-g} W_{(E)\mu\nu\alpha\beta} W_{(E)}^{\mu\nu\alpha\beta}$$

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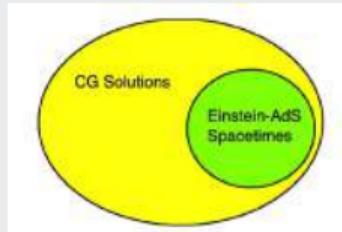
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Conformal Renormalization

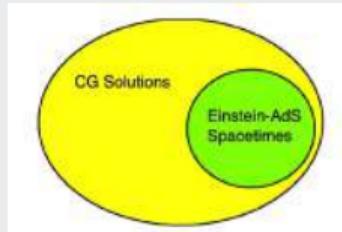
Embedding Einstein theory in Conformal Gravity



- Why?: Conformal Gravity is finite for AAdS conditions. [Grumiller et al., 2013]
- What for?: Renormalization should be inherited by the Einstein sector.
- How?: a *holographic* mechanism to turn CG into Einstein

Conformal Renormalization

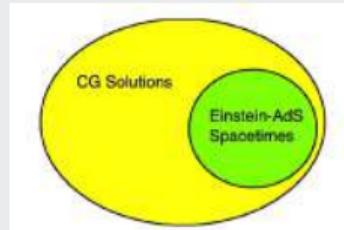
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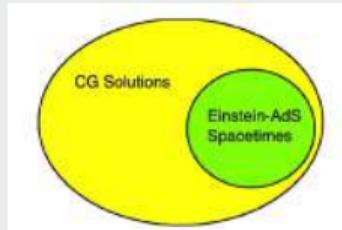
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Einstein Gravity from Conformal Gravity in 4D

Einstein gravity from CG with Neumann bc's [Maldacena, 2011]

$$I_{CG} = \alpha_{CG} \int_M d^4x \sqrt{-g} W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta}$$

Conformal Gravity in 4D with boundary conditions

$$ds^2 = \frac{1}{z^2} dz^2 + \frac{1}{z^2} g_{ij}(x, z) dx^i dx^j, \quad g_{ij}(x, \rho) = g_{00,0}(x) + z^2 g_{00,1}(x) + \dots$$

$$\partial_z g_{00,0}(x) = 0, \quad \partial_z g_{00,1}(x) = 0, \dots$$

$$R_{\mu\nu} = 2 D_{\mu\nu} + z^2 D_{\mu\nu,0} = 0,$$

$$D_{\mu\nu} = \nabla_{\mu}\nabla_{\nu} - \nabla_{\nu}\nabla_{\mu}$$

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Conformal Gravity in 4D: $\mathcal{L}_{CG} = \frac{1}{2} R_{\mu\nu} g^{\mu\nu} - \frac{1}{2} \lambda R^2$

$$ds^2 = \frac{1}{r^2} dt^2 + \frac{1}{r^2} g_{ij}(x, r) dx^i dx^j, \quad g_{ij}(x, r) = g_{00}(r) + r^2 g_{ij}(x) + \dots$$
$$\partial_r g_{00}(r) = 0, \quad \partial_r g_{ij}(x) = 0$$

$$R_{\mu\nu} = \frac{1}{r^2} \partial_r^2 g_{\mu\nu} + \frac{2}{r^3} \partial_r g_{\mu\nu} + \frac{1}{r^2} g_{\mu\nu} \Delta g(r)$$

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Fefferman-Graham expansion for AAdS spaces in CG

$$ds^2 = \frac{\ell^2}{z^2} dz^2 + \frac{1}{z^2} g_{ij}(x, z) dx^i dx^j, \quad g_{ij}(x, \rho) = g_{(0)ij}(x) + z^2 g_{(2)ij}(x) + \dots \\ + z g_{(1)ij}(x) + \dots$$

$$B_{\mu\nu} = \nabla^\lambda \Omega_{\mu\nu\lambda} + \nabla^\lambda \Omega_{\nu\mu\lambda} = 0, \quad \Omega_{\mu\nu\lambda} = \nabla_\mu \Omega_{\nu\lambda} - \nabla_\nu \Omega_{\mu\lambda}$$

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Based on: [arXiv:1105.5069](https://arxiv.org/abs/1105.5069) by Maldacena et al.

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EOM for Conformal Gravity

$$B_{\mu\nu} = \nabla^\lambda C_{\mu\nu\lambda} + S^{\lambda\sigma} W_{\lambda\mu\sigma\nu} = 0, \quad C^\mu_{\nu\lambda} = \nabla_\nu S^\mu_\lambda - \nabla_\lambda S^\mu_\nu$$

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Einstein spaces: holographic prescription

Einstein-AdS spaces

$$S_\nu^\mu = -\frac{1}{2\ell^2}\delta_\nu^\mu, \quad C_{\mu\nu\lambda} = 0, \quad B_{\mu\nu} = 0$$

Conformal gauge

$$\mathcal{D}_j^3 = \mathcal{D}_j^2 + \frac{1}{\ell^2} \mathcal{D}_j^1 = 0$$

$$H_{\mu\nu} = 0 \iff \partial_z g_{ij} = g_{(1)ij} = 0 \text{ and } \text{tr}(\partial_z^3 g_{ij}) \sim \text{tr}(g_{(3)ij}) = 0$$

$g_{(2)ij}$ is free data in CG \implies chosen as in Einstein $g_{(2)ij} = -\ell^2 S_{(0)ij}$
[Imbimbo, Schwimmer, Theisen and Yankielowicz, hep-th/9910267]

Conformal boundary conditions \implies boundary conditions for $S_{(0)ij}$

$$\text{Im}[E] = \text{Im}$$

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Traceless Ricci tensor

$$H_\nu^\mu = R_\nu^\mu - \frac{1}{D}R\delta_\nu^\mu = 0$$

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CG action for Einstein spaces = Renormalized Einstein-AdS action

$$I_{\text{CG}}[E] = I_{\text{HR}}$$

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AdS gravity in 6D

EH Action+Euler term

$$\tilde{I}_{\text{ren}} = \frac{1}{16\pi G} \int_M d^6x \sqrt{-g} \left(R + \frac{20}{\ell^2} - \frac{\ell^4}{72} (\text{Euler})_6 \right),$$

Integration by parts and boundary terms

$$\begin{aligned} \tilde{I}_{\text{ren}} &= \frac{1}{16\pi G \times 192} \int_M d^6x \sqrt{-g} \delta_{[\mu\nu\rho\sigma\tau\lambda]}^{[\mu\nu\rho\sigma\tau\lambda]} [R^{\mu\nu\rho\sigma} \delta_{[\mu\nu\rho\sigma]}^{[\mu\nu\rho\sigma]} \delta_{[\tau\lambda]}^{[\tau\lambda]} \\ &\quad + \frac{2}{3} \delta_{[\mu\nu\rho\sigma]}^{[\mu\nu\rho\sigma]} \delta_{[\tau\lambda]}^{[\tau\lambda]} - \frac{2}{3} \delta_{[\mu\nu\rho\sigma\tau\lambda]}^{[\mu\nu\rho\sigma\tau\lambda]} \delta_{[\mu\nu\rho\sigma\tau\lambda]}^{[\mu\nu\rho\sigma\tau\lambda]}]. \end{aligned}$$

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Renormalized action in 6D

$$\tilde{I}_{\text{ren}} = \frac{1}{16\pi G \times 192} \int_M d^6x \sqrt{-g} \text{e}^{-\frac{20}{\ell^2 g_{\mu\nu} g_{\rho\sigma}}} \left[R g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho} \right] \frac{1}{g^{\mu\nu} g^{\rho\sigma}} \left[\frac{1}{2} g^{\mu\rho} g^{\nu\sigma} \partial_\rho \partial_\sigma \ln g_{\mu\nu} - \frac{1}{2} g^{\mu\nu} g^{\rho\sigma} \partial_\rho \partial_\sigma \ln g_{\mu\nu} \right]$$

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In terms of fully-antisymmetric objects

$$\begin{aligned} \tilde{I}_{\text{ren}} &= \frac{1}{16\pi G \times 192} \int_M d^6x \sqrt{-g} \delta_{[\mu_1 \dots \mu_6]}^{[\nu_1 \dots \nu_6]} [R_{\nu_1 \nu_2}^{\mu_1 \mu_2} \delta_{[\nu_3 \nu_4]}^{[\mu_3 \mu_4]} \delta_{[\nu_5 \nu_6]}^{[\mu_5 \mu_6]} \\ &\quad + \frac{2}{3\ell^2} \delta_{[\nu_1 \nu_2]}^{[\mu_1 \mu_2]} \delta_{[\nu_3 \nu_4]}^{[\mu_3 \mu_4]} \delta_{[\nu_5 \nu_6]}^{[\mu_5 \mu_6]} - \frac{\ell^4}{3} R_{\nu_1 \nu_2}^{\mu_1 \mu_2} R_{\nu_3 \nu_4}^{\mu_3 \mu_4} R_{\nu_5 \nu_6}^{\mu_5 \mu_6}], \end{aligned}$$

AdS gravity in 6D

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AdS gravity in 6D

Polynomial of $W_{(E)}$

$$\begin{aligned}\tilde{I}_{\text{ren}} &= \frac{\ell^4}{16\pi G \times 4!} \int_M d^6x \sqrt{-g} \left[\frac{1}{2\ell^2} \delta_{[\mu_1 \dots \mu_4]}^{[\nu_1 \dots \nu_4]} W_{(E)\nu_1\nu_2}^{\mu_1\mu_2} W_{(E)\nu_3\nu_4}^{\mu_3\mu_4} \right. \\ &\quad \left. - \frac{1}{4!} \delta_{[\mu_1 \dots \mu_6]}^{[\nu_1 \dots \nu_6]} W_{(E)\nu_1\nu_2}^{\mu_1\mu_2} W_{(E)\nu_3\nu_4}^{\mu_3\mu_4} W_{(E)\nu_5\nu_6}^{\mu_5\mu_6} \right],\end{aligned}$$

Conformal Gravity in 6D

There are three Conformal Invariants in 6D

$$I_1 = W_{\alpha\beta\mu\nu} W^{\alpha\sigma\lambda\nu} W_{\sigma}^{\beta\mu}_{\lambda},$$

$$I_2 = W_{\mu\nu\alpha\beta} W^{\alpha\beta\sigma\lambda} W_{\sigma\lambda}^{\mu\nu},$$

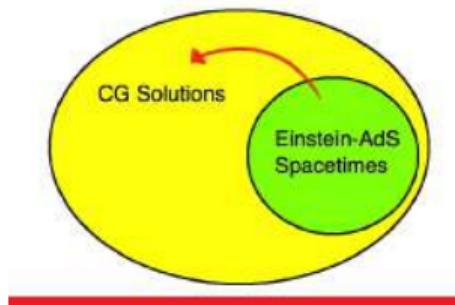
$$I_3 = W_{\mu\rho\sigma\lambda} \left(\delta_{\nu}^{\mu} \square + 4R_{\nu}^{\mu} - \frac{6}{5} R \delta_{\nu}^{\mu} \right) W^{\nu\rho\sigma\lambda} + \nabla_{\mu} J^{\mu},$$

with

$$\begin{aligned} J_{\mu} &= 4R_{\mu}^{\lambda\rho\sigma} \nabla^{\nu} R_{\nu\lambda\rho\sigma} + 3R^{\nu\lambda\rho\sigma} \nabla_{\mu} R_{\nu\lambda\rho\sigma} - 5R^{\nu\lambda} \nabla_{\mu} R_{\nu\lambda} \\ &\quad + \frac{1}{2} R \nabla_{\mu} R - R_{\mu}^{\nu} \nabla_{\nu} R + 2R^{\nu\lambda} \nabla_{\nu} R_{\lambda\mu}. \end{aligned}$$

Conformal Covariantization

Einstein	\rightarrow	Conformal	Cl's
$\delta_{[\mu_1 \dots \mu_6]}^{[\nu_1 \dots \nu_6]} \mathbf{W}_{(E)\nu_1\nu_2}^{\mu_1\mu_2} \mathbf{W}_{(E)\nu_3\nu_4}^{\mu_3\mu_4} \mathbf{W}_{(E)\nu_5\nu_6}^{\mu_5\mu_6}$	\rightarrow	$\delta_{[\mu_1 \dots \mu_6]}^{[\nu_1 \dots \nu_6]} \mathbf{W}_{\nu_1\nu_2}^{\mu_1\mu_2} \mathbf{W}_{\nu_3\nu_4}^{\mu_3\mu_4} \mathbf{W}_{\nu_5\nu_6}^{\mu_5\mu_6}$	$32(2\mathbf{I}_1 + \mathbf{I}_2)$
$-\frac{1}{\ell^2} \delta_{[\mu_1 \dots \mu_4]}^{[\nu_1 \dots \nu_4]} \mathbf{W}_{(E)\nu_1\nu_2}^{\mu_1\mu_2} \mathbf{W}_{(E)\nu_3\nu_4}^{\mu_3\mu_4}$	\rightarrow	$\delta_{[\mu_1 \dots \mu_5]}^{[\nu_1 \dots \nu_5]} \mathbf{W}_{\nu_1\nu_2}^{\mu_1\mu_2} \mathbf{W}_{\nu_3\nu_4}^{\mu_3\mu_4} \mathbf{S}_{\nu_5}^{\mu_5} + 16\mathbf{C}^{\mu\nu\lambda} \mathbf{C}_{\mu\nu\lambda} + \nabla^\mu \mathbf{J}_\mu$	$4\mathbf{I}_1 - \mathbf{I}_2 - \mathbf{I}_3$
$\mathbf{J}_\mu = 16\mathbf{W}_\mu^{\kappa\lambda\nu} \mathbf{C}_{\kappa\lambda\nu} - 2\mathbf{W}_{\nu\sigma}^{\kappa\lambda} \nabla_\mu \mathbf{W}_{\kappa\lambda}^{\nu\sigma}$			



Lu-Pang-Pope CG in 6D

6D CG with an Einstein sector [Lu, Pang and Pope, 2013]

$$I_{CG} = \alpha_{CG} \int_M d^6x \sqrt{-\hat{g}} \left(\frac{1}{4!} \delta^{[\nu_1 \dots \nu_6]}_{[\mu_1 \dots \mu_6]} W^{\mu_1 \mu_2}_{\nu_1 \nu_2} W^{\mu_3 \mu_4}_{\nu_3 \nu_4} W^{\mu_5 \mu_6}_{\nu_5 \nu_6} + \frac{1}{2} \delta^{[\nu_1 \dots \nu_5]}_{[\mu_1 \dots \mu_5]} W^{\mu_1 \mu_2}_{\nu_1 \nu_2} W^{\mu_3 \mu_4}_{\nu_3 \nu_4} S^{\mu_5}_{\nu_5} \right. \\ \left. + 8C^{\mu\nu\lambda} C_{\mu\nu\lambda} \right) + \alpha_{CG\partial M} d^5x \sqrt{-h} n^\mu \left(8W_\mu^{\kappa\lambda\nu} C_{\kappa\lambda\nu} - W_{\nu\sigma}^{\kappa\lambda} \nabla_\mu W_{\kappa\lambda}^{\nu\sigma} \right).$$

- LPP action appears as type-B anomaly and one-loop divergences in 7D
- Variation of I_{CG} gives EOM in terms of Weyl, Cotton and Schouten tensors.
- Any Einstein-AdS spacetime is solution of LPP CG. [Anastasiou, Araya and RO, 2010.15146]

Lu-Pang-Pope CG in 6D

6D CG with an Einstein sector [Lu, Pang and Pope, 2013]

$$I_{CG} = \alpha_{CG} \int_M d^6x \sqrt{-\hat{g}} \left(\frac{1}{4!} \delta^{[\nu_1 \dots \nu_6]}_{[\mu_1 \dots \mu_6]} W^{\mu_1 \mu_2}_{\nu_1 \nu_2} W^{\mu_3 \mu_4}_{\nu_3 \nu_4} W^{\mu_5 \mu_6}_{\nu_5 \nu_6} + \frac{1}{2} \delta^{[\nu_1 \dots \nu_5]}_{[\mu_1 \dots \mu_5]} W^{\mu_1 \mu_2}_{\nu_1 \nu_2} W^{\mu_3 \mu_4}_{\nu_3 \nu_4} S^{\mu_5}_{\nu_5} \right. \\ \left. + 8C^{\mu\nu\lambda} C_{\mu\nu\lambda} \right) + \alpha_{CG\partial M} d^5x \sqrt{-h} n^\mu \left(8W_\mu^{\kappa\lambda\nu} C_{\kappa\lambda\nu} - W_{\nu\sigma}^{\kappa\lambda} \nabla_\mu W_{\kappa\lambda}^{\nu\sigma} \right).$$

- LPP action appears as type-B anomaly and one-loop divergences in 7D
- Variation of I_{CG} gives EOM in terms of Weyl, Cotton and Schouten tensors.
- Any Einstein-AdS spacetime is solution of LPP CG. [Anastasiou, Araya and RO, 2010.15146]

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Lu-Pang-Pope CG in 6D

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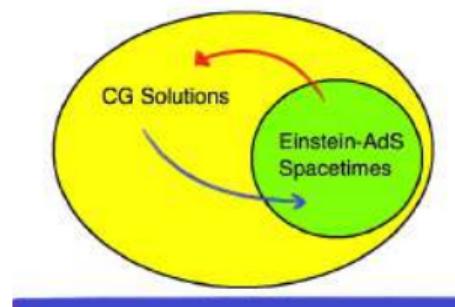
$$I_{CG} = \alpha_{CG} \int_M d^6x \sqrt{-\hat{g}} \left(\frac{1}{4!} \delta_{[\mu_1 \cdots \mu_6]}^{[\nu_1 \cdots \nu_6]} W_{\nu_1 \nu_2}^{\mu_1 \mu_2} W_{\nu_3 \nu_4}^{\mu_3 \mu_4} W_{\nu_5 \nu_6}^{\mu_5 \mu_6} + \frac{1}{2} \delta_{[\mu_1 \cdots \mu_5]}^{[\nu_1 \cdots \nu_5]} W_{\nu_1 \nu_2}^{\mu_1 \mu_2} W_{\nu_3 \nu_4}^{\mu_3 \mu_4} S_{\nu_5}^{\mu_5} \right. \\ \left. + 8C^{\mu\nu\lambda} C_{\mu\nu\lambda} \right) + \alpha_{CG\partial M} d^5x \sqrt{-h} n^\mu \left(8W_\mu^{\kappa\lambda\nu} C_{\kappa\lambda\nu} - W_{\nu\sigma}^{\kappa\lambda} \nabla_\mu W_{\kappa\lambda}^{\nu\sigma} \right).$$

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And back... (Einstein gravity from CG in 6D)

LPP CG action decomposed into Einstein and non-Einstein parts:

$$I_{CG} = -4! \alpha_{CG} \int_M d^6x \sqrt{-g} [P_6(W_{(E)}) + Q(W_{(E)}, H)] \\ - \alpha_{CG} \int_{\partial M} d^5x \sqrt{-h} n^\mu \left(W_{(E)\nu\sigma}^{\kappa\lambda} \nabla_\mu W_{(E)\kappa\lambda}^{\nu\sigma} \right).$$



Back to Einstein gravity (with an extra term)

Einstein condition, and $\alpha_E = -\frac{\ell^4}{384\pi G}$:

$$I_{CG}[E] = \frac{1}{16\pi G} \int_M d^6x \sqrt{-g} \left(R + \frac{20}{\ell^2} - \frac{\ell^4}{72} (Euler)_6 \right) + \frac{\ell^4}{384\pi G} \int_{\partial M} d^5x \sqrt{-h} n^\mu \left(W_{(E)\nu\sigma}^{\kappa\lambda} \nabla_\mu W_{(E)\kappa\lambda}^{\nu\sigma} \right),$$

Conformal transformation:

$$\mathcal{N} = \frac{\ell^2}{16\pi G} \int d^6x \sqrt{-R} W^{ijkl} (Ricci_{ijkl}) + \dots$$

Equation for Einstein gravity = Conformal Einstein equation

$$Im[E] = Im$$

Back to Einstein gravity (with an extra term)

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Conformal factor ϕ

$$\mathcal{N} = \frac{\ell^4}{16\pi G} \int d^6x \sqrt{-g} R^{00} \phi^6 (R_{00})^2 + \dots$$

Conformal Einstein gravity = Standard Einstein gravity

$$Im[B] = Im$$

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Performing asymptotic expansions

$$\Delta I = \frac{\ell^3}{192\pi G} \int_{\partial M} d^5x \sqrt{-h} \mathcal{W}^{ijkl}(h) \mathcal{W}_{ijkl}(h) + \dots$$

Correction for Einstein gravity → Generalized Einstein's equations

$$Int[E] = Int$$

Back to Einstein gravity (with an extra term)

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Conformal Covariant operator \rightarrow Conformal Functionals

$$\text{Im}[E] = \text{Im}$$

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CG action for Einstein spaces = Renormalized Einstein-AdS action

$$I_{CG}[E] = I_{HR}$$

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CG action for Einstein spaces = Renormalized Einstein-AdS action

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Curvature-squared terms and conical defects

Riemann squared term

$$\int_{M^{(\alpha)}} d^4x \sqrt{g} \left(Rie^{(\alpha)} \right)^2 = \int_M d^4x \sqrt{g} Rie^2 + 8\pi(1-\alpha) \int_{\Sigma} d^2y \sqrt{\gamma} \left(R_{ABAB} - \mathcal{K}_{mn}^{(A)} \mathcal{K}_{(A)}^{mn} \right) + \dots$$

Scalar curvature term

$$\int_{M^{(\alpha)}} d^4x \sqrt{g} \left(Ric^{(\alpha)} \right)^2 = \int_M d^4x \sqrt{g} Ric^2 + 8\pi(1-\alpha) \int_{\Sigma} d^2u \sqrt{\gamma} \left(R_{AA} - \frac{1}{2} \mathcal{K}^{(A)} \mathcal{K}_{(A)} \right) + \dots$$

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Curvature-squared terms and conical defects

Riemann squared term

$$\int_{M^{(\alpha)}} d^4x \sqrt{g} \left(Rie^{(\alpha)} \right)^2 = \int_M d^4x \sqrt{g} Rie^2 + 8\pi(1-\alpha) \int_{\Sigma} d^2y \sqrt{\gamma} \left(R_{ABAB} - \mathcal{K}_{mn}^{(A)} \mathcal{K}_{(A)}^{mn} \right) + \dots$$

Conformal term

$$\int_{M^{(\alpha)}} d^4x \sqrt{g} \left(R^{(\alpha)} \right)^2 = \int_M d^4x \sqrt{g} R^2 + 8\pi(1-\alpha) \int_{\Sigma} d^2y \sqrt{\gamma} \left(R_{AA} - \frac{1}{2} R^{(A)} R_{(A)} \right) + \dots$$

Scalar term

$$\int_{M^{(\alpha)}} d^4x \sqrt{g} \left(R^{(\alpha)} \right)^2 = \int_M d^4x \sqrt{g} R^2 + 8\pi(1-\alpha) \int_{\Sigma} \sqrt{\gamma} R + \dots$$

[Fursaev-Patrushев-Solodukhin, 2013]

Curvature-squared terms and conical defects

Riemann squared term

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Ricci squared term

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Curvature-squared terms and conical defects

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[Fursaev-Patrushев-Solodukhin, 2013]

Curvature-squared terms and conical defects

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Curvature-squared terms and conical defects

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[Fursaev-Patrushев-Solodukhin, 2013]

Conformal Gravity and Conical Defects

4D Conformal Gravity

$$I_{CG} = \frac{\ell^2}{64\pi G} \int_M d^4x \sqrt{-g} W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta} - \frac{\pi\ell^2}{2G} \chi[M]$$

Conformal gravity

$$I_{CG} = \frac{\ell^2}{64\pi G} \int d^4x \sqrt{g} \left(R\omega^2 - 2R\omega^2 + \frac{1}{3} R^2 \right) - \frac{\pi\ell^2}{2G} \chi[M]$$

Conformal coordinate condition

$$\int d^4x \sqrt{g} |W|^2 = \int d^4x \sqrt{g} |W|^2 + 8\pi(1-\omega) \int d^4x \sqrt{g} K_S + o(\omega - 1)$$

Conformal Gravity and Conical Defects

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$$ds^2 = \frac{dr^2}{r^2 \cos^2\theta} + r^2 d\theta^2 + r^2 \sin^2\theta (dr^2 - 2rd\theta dr + \frac{1}{3}d\theta^2) - \frac{r^2}{3} d\phi^2$$

$$\int d^4x \sqrt{g} R^2 = \int d^4x \sqrt{g} R^2 + \delta\omega(1-\omega) \int d^4x \sqrt{g} K_S + \mathcal{O}(\omega - 1)$$

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Conformal gravity is conformally invariant

$$\int d^4x \sqrt{g} |W^{(0)}|^2 = \int d^4x \sqrt{g} |W|^2 + 8\pi(1-\omega) \int d^4x \sqrt{g} K_S + \mathcal{O}(\alpha - \omega)^2$$

Conformal Gravity and Conical Defects

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In a manifold with a conical singularity

$$\int_{M^{(\alpha)}} d^4x \sqrt{g} \left| W^{(\alpha)} \right|^2 = \int_M d^4x \sqrt{g} |W|^2 + 8\pi(1-\alpha) \int_{\Sigma} d^2y \sqrt{\gamma} K_{\Sigma} + \mathcal{O}\left((1-\alpha)^2\right)$$

Conformal Gravity and Conical Defects

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In a manifold with a conical singularity

$$\int_{M^{(\alpha)}} d^4x \sqrt{g} \left| W^{(\alpha)} \right|^2 = \int_M d^4x \sqrt{g} |W|^2 + 8\pi(1-\alpha) \int_{\Sigma} d^2y \sqrt{\gamma} K_{\Sigma} + \mathcal{O}\left((1-\alpha)^2\right)$$

Conformal Gravity and Conical Defects

Conformal Invariant in codimension-2 (Graham-Witten anomaly)

$$K_{\Sigma} = W_{mn}^{mn} - P_{mn}^{(A)} P_{(A)}^{mn},$$

Traceless part of the extrinsic curvature

$$P_{mn}^{(A)} = \mathcal{K}_{mn}^{(A)} - \frac{1}{2} \mathcal{K}^{(A)} \gamma_{mn}$$

Conformal factor α

$$I_{CG}^{(\alpha)} = I_{CG} + \frac{(1-\alpha)}{4G} L_{\Sigma} + \mathcal{O}((1-\alpha)^2)$$

Conformal factor α and the conformal anomaly

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$$T_{\mu\nu}^{(A)} = T_{\mu\nu} + \frac{(1-\alpha)}{4G}L_{\mu\nu} + \mathcal{O}((\alpha-\alpha')^2)$$

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Conformal Gravity Action in codimension-2

Conformal Gravity and Conical Defects

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Conformal Invariance in Codimension-2 is inherited from the Bulk!

Renormalized Area

Minimal surface/Einstein ambient space

$$\mathcal{A}_{\text{ren}} = L_{\Sigma_{\text{min}}} [E]$$

$$L_{\Sigma_{\text{min}}} [\Sigma] = \frac{\ell}{2} \int d^2y \sqrt{g} \left[g_{\mu\nu} (\partial^\mu E_{\nu\rho} - \partial_\nu^A E_{\mu\rho}) - 2\pi l^2 \chi [\Sigma] \right]$$

Conformal renormalization of the area functional

$$L_{\Sigma_{\text{min}}} [\Sigma] = \frac{\ell}{2} \int d^2y \sqrt{g} R_{\mu\nu} \left(E_{\mu}^{\mu\nu} + \frac{1}{\rho} \delta_{\mu}^{\mu\nu} \right) - 2\pi l^2 \chi [\Sigma]$$

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$$\mathcal{A}_{\text{ren}} = L_{\Sigma_{\text{min}}} [E]$$

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$$L_{\text{min}} [\Sigma] = \frac{\ell}{2} \int d^2x \sqrt{g_{\mu\nu}} \left(R_{\mu\nu}^{(D)} + \frac{1}{\ell^2} \delta_{\mu\nu}^{(D)} \right) - 2\pi l^2 \chi [\Sigma]$$

Renormalized Area

Minimal surface/Einstein ambient space

$$\mathcal{A}_{\text{ren}} = L_{\Sigma_{\text{min}}} [E]$$

Renormalized Area

$$\mathcal{A}_{\text{ren}} [\Sigma] = \frac{\ell^2}{2} \int_{\Sigma} d^2y \sqrt{\gamma} \left[W_{(\mathbb{E})mn}^{mn} - P_{mn}^{(A)} P_{(A)}^{mn} \right] - 2\pi\ell^2 \chi [\Sigma]$$

Renormalized area in terms of the metric components and the Ricci tensor

$$\mathcal{A}_{\text{ren}} [\Sigma] = \frac{\ell^2}{4} \int_{\Sigma} d^2y \sqrt{\delta} \delta_{mn}^{pq} \left(R_{pq}^{mn} + \frac{1}{\ell^2} \delta_{pq}^{mn} \right) - 2\pi\ell^2 \chi [\Sigma]$$

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$$\mathcal{A}_{\text{ren}}[\Sigma] = \frac{\ell^2}{2} \int_{\Sigma} d^2y \sqrt{\gamma} R_{mn} \left(R_{mn}^{mn} + \frac{1}{\ell^2} \delta_{mn}^{mn} \right) - 2\pi\ell^2\chi[\Sigma]$$

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Ren. Area [Alexakis-Mazzeo, 2010] / Ren. HEE [Anastasiou-Araya-RØ, 2018]

$$\mathcal{A}_{\text{ren}}[\Sigma] = \frac{\ell^2}{4} \int_{\Sigma} d^2y \sqrt{\gamma} \delta_{mn}^{pq} \left(\mathcal{R}_{pq}^{mn} + \frac{1}{\ell^2} \delta_{pq}^{mn} \right) - 2\pi\ell^2\chi[\Sigma]$$

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$$\mathcal{A}_{\text{ren}} = L_{\Sigma_{\min}}[E]$$

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Willmore Energy

Functional defined on compact and orientable 2D surfaces immersed in \mathbb{R}^3



Intrinsic energy of the surface

$$W = \int d\sigma g^{ij} R_{ij}$$

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Functional defined on compact and orientable 2D surfaces immersed in \mathbb{R}^3



WILLMORE ENERGY

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In terms of the (spatial) mean curvature

$$\mathcal{W}[\Sigma] = \int_{\Sigma} d^2y \sqrt{\gamma} H^2$$

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In terms of the (spatial) mean curvature

$$\mathcal{W}[\Sigma] = \int_{\Sigma} d^2y \sqrt{\gamma} H^2$$

Willmore Energy from Renormalized Area

Renormalized Area

$$\mathcal{A}_{\text{ren}} [\Sigma] = \frac{\ell^2}{2} \int_{\Sigma} d^2y \sqrt{\gamma} \left(W_{(\mathbb{E})mn}^{mn} - P_{mn}^{(A)} P_{(A)}^{mn} \right) - 2\pi\ell^2 \chi [\Sigma]$$

Boundary conditions: $\partial_\nu \gamma = 0$, $\partial_\nu \chi = 0$

$$W = 0 \quad F^{ij} = 0$$

Conformal condition:

$$\mathcal{A}_{\text{ren}} [\Sigma] = \frac{\ell^2}{2} \int_{\Sigma} d^2y \sqrt{\gamma} \left(R_g - R + 2H^2 \right) - 2\pi\ell^2 \chi [\Sigma]$$

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Renormalized Area

$$\mathcal{A}_{\text{ren}} [\Sigma] = \frac{\ell^2}{2} \int_{\Sigma} d^2y \sqrt{\gamma} \left(W_{(\mathbb{E})mn}^{mn} - P_{mn}^{(A)} P_{(A)}^{mn} \right) - 2\pi\ell^2 \chi [\Sigma]$$

where \mathbb{E} is the Willmore operator and $P_{mn}^{(A)}$ is the projector onto the AdS boundary.

$$W = 0 \quad P^{(A)} = 0$$

Willmore energy

$$\mathcal{A}_{\text{ren}} [\Sigma] = \frac{\ell^2}{2} \int_{\Sigma} d^2y \sqrt{\gamma} (R^2 - R + 2R^2) - 2\pi\ell^2 \chi [\Sigma]$$

Willmore Energy from Renormalized Area

Renormalized Area

$$\mathcal{A}_{\text{ren}} [\Sigma] = \frac{\ell^2}{2} \int_{\Sigma} d^2y \sqrt{\gamma} \left(W_{(\mathbf{E})mn}^{mn} - P_{mn}^{(A)} P_{(A)}^{mn} \right) - 2\pi\ell^2 \chi [\Sigma]$$

For pure/global AdS_4 as ambient space, constant-time slice

$$W = 0 \quad \mathcal{K}^{(t)} = 0$$

Willmore energy

$$\mathcal{A}_{\text{ren}} [\Sigma] = \frac{\ell^2}{2} \int d^2y \sqrt{\gamma} \left(R_g^2 - R + 2R^2 \right) - 2\pi\ell^2 \chi [\Sigma]$$

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For pure/global AdS_4 as ambient space, constant-time slice

$$W = 0 \quad \mathcal{K}^{(t)} = 0$$

Renormalized area

$$\mathcal{A}_{\text{ren}} [\Sigma] = -\frac{\ell^2}{2} \int_{\Sigma} d^2y \sqrt{\gamma} \left(R^2 - R + 2R^2 - 2\pi\ell^2 \chi [\Sigma] \right)$$

Willmore Energy from Renormalized Area

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Gauss-Codazzi relations

$$\mathcal{A}_{\text{ren}} [\Sigma] = -\frac{\ell^2}{2} \int_{\Sigma} d^2 y \sqrt{\gamma} \left(R_{ij}^{ij} - \mathcal{R} + 2H^2 \right) - 2\pi\ell^2 \chi [\Sigma]$$

Willmore Energy from Renormalized Area

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Willmore Energy

In the conformal frame $\hat{g}_{\mu\nu}$

$$\mathcal{A}_{\text{ren}}[\Sigma] = \frac{\ell^2}{2} \int_{\Sigma} d^2y \sqrt{\hat{\gamma}} \left(\hat{\mathcal{R}} - 2\hat{H}^2 \right) - 2\pi\ell^2\chi[\Sigma]$$

For a compact surface

$$\int_{\Sigma_{\text{bound}}} d^2y \sqrt{\hat{\mathcal{R}}} = 4\pi\chi[\Sigma_{\text{bound}}]$$

Willmore energy: $\mathcal{W}[\Sigma_{\text{bound}}] = \mathcal{A}_{\text{ren}}[\Sigma_{\text{bound}}]$

$$\mathcal{A}_{\text{ren}}[\Sigma_{\text{bound}}] = -\ell^2\mathcal{W}[\Sigma_{\text{bound}}]$$

Willmore Energy

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Conformal Willmore energy

$$\int d^2y \sqrt{\hat{\gamma}} (\hat{\mathcal{R}} - 4\pi\chi[\Sigma_{\text{conf}}])$$

Willmore energy: $\mathcal{A}_{\text{ren}} [\Sigma_{\text{conf}}] = -\ell^2 V[\Sigma_{\text{conf}}]$

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For a compact surface

$$\int_{\Sigma_{\text{comp}}} d^2y \sqrt{\hat{\gamma}} \hat{\mathcal{R}} = 4\pi\chi[\Sigma_{\text{comp}}]$$

Willmore energy: $\mathcal{A}_{\text{ren}} [\Sigma_{\text{comp}}] = -\ell^2\mathcal{W}[\Sigma_{\text{comp}}]$

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Willmore Energy

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For a compact surface

$$\int_{\Sigma_{\text{comp}}} d^2y \sqrt{\hat{\gamma}} \hat{\mathcal{R}} = 4\pi\chi[\Sigma_{\text{comp}}]$$

Willmore energy for a compact surface

$$A_{\text{ren}}[\Sigma_{\text{comp}}] = -\ell^2\chi[\Sigma_{\text{comp}}]$$

Willmore Energy

In the conformal frame $\hat{g}_{\mu\nu}$

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$$\int_{\Sigma_{\text{comp}}} d^2y \sqrt{\hat{\gamma}} \hat{\mathcal{R}} = 4\pi\chi[\Sigma_{\text{comp}}]$$

Willmore Energy [Anastasiou, Moreno, RØ, Rivera-Betancour, 2020]

$$\mathcal{A}_{\text{ren}} [\Sigma_{\text{comp}}] = -\ell^2 \mathcal{W} [\Sigma_{\text{comp}}]$$

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$$\mathcal{A}_{\text{ren}} [\Sigma_{\text{comp}}] = -\ell^2 \mathcal{W} [\Sigma_{\text{comp}}]$$

Bonus Track: Reduced Hawking Mass

Arbitrary Σ , Einstein ambient space

$$L_\Sigma [E] = \frac{\ell^2}{4} I_H [\Sigma] - 2\pi\ell^2 \chi [\Sigma]$$

$$I_H [\Sigma] = 2 \int d^2\theta \sqrt{g} \left[R + \frac{2}{\beta} - \frac{1}{2} \left(\kappa^{(4)} \right)^2 \right]$$

$$L_\Sigma [E] = A_{\text{ext}} [\Sigma] - \frac{\ell^2}{4} \int d^2\theta \sqrt{g} \left(\kappa^{(4)} \right)^2$$

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Reduced Hawking Mass I_H [Fischetti and Wiseman, 2016]

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Generalizes Renormalized Area Functional

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Outlook

Conformal Invariance in the Bulk \implies Conformal Invariance in Codimension-2

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Renormalized Volume \implies Renormalized Area (in conically singular manifolds)

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